

# The Fundamental Constants in Physics and their Time Dependence

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## Abstract

We discuss the fundamental constants in the Standard Model of particle physics, in particular possible changes of these constants on the cosmological time scale. The Grand Unification of the observed strong, electromagnetic and weak interactions implies relations between time variations of the finestructure constant  $\alpha$  and the QCD scale  $\Lambda_c$ . The astrophysical observation of a variation of  $\alpha$  implies a time variation of  $\Lambda_c$  of the order of at least  $10^{-15}/year$ . Several experiments in Quantum Optics, which were designed to look for a time variation of  $\Lambda_c$ , are discussed.

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## 1 The Standard Model

The Standard Model of particle physics consists of

- a) the theory of the strong interactions (QCD)[1],
- b) the theory of the electroweak interactions, based on the gauge group  $SU(2) \times U(1)$ [2].

The theory of QCD is an unbroken gauge theory, based on the gauge group  $SU(3)$ , acting in the internal space of "color". The basic fermions of the theory are the various quarks, which form color triplets. The 8 massless gauge bosons are  $SU(3)$ -octets, the gluons. The interactions of the quarks and gluons are dictated by the gauge properties of the theory. The quarks and gluons interact through the vertex  $g_s \cdot \bar{q} \gamma_\mu \frac{\lambda_i}{2} q \cdot A_i^\mu$ , where  $q$  are the quark fields and  $A_i^\mu$  the eight gluon fields. The eight  $SU(3)$ -matrices are denoted by  $\lambda_i$ . The strength of the coupling is given by the coupling constant  $g_s$ .

QCD is a non-Abelian gauge theory. This implies that there is a direct coupling of the gluons among each other. There is a trilinear coupling, proportional to  $g_s$ , and a quadrilinear coupling, proportional

to  $g_s^2$ . It is assumed, but thus far not proven, that the QCD gauge interaction leads to a confinement of all colored quanta, in particular of the quarks and the gluons. Replacing the continuous space–time continuum by a lattice, one can solve the QCD field equations with the computer. The results confirm the confinement hypothesis, but it remains unclear whether one can perform the limit, in which the lattice spacing goes to zero, without changing the result.

The experiments strongly support the QCD theory. It has the property of asymptotic freedom: the strength of the interactions goes to zero on a logarithmic scale at high energies. At low energies the interaction gets large, and the confinement property might indeed be true. The equations, describing the renormalization of the coupling constant, give for  $\alpha_s = \frac{g_s^2}{4\pi}$ :

$$\begin{aligned}\mu \cdot \frac{\partial \alpha_s}{\partial \mu} &= -\frac{\beta_0}{2\pi} \alpha_s^2 - \frac{\beta_1}{4\pi^2} \alpha_s^3 - \dots \\ \beta_0 &= 11 - \frac{2}{3} n_f \\ \beta_1 &= 51 - \frac{19}{3} n_f\end{aligned}\tag{1}$$

( $n_f$ : number of quark flavors with mass less than the energy scale  $\mu$  ).

Since the interactions gets weak at high energies, the quarks and also the gluons appear nearly as pointlike objects at small distances. This has been observed in the experiments of deep inelastic scattering of electrons, myons and neutrinos off nuclear targets.

Since the strong coupling constant at high energies is small, but not zero, one expects small departures from the pointlike structure, i. e. violations of the scaling behaviour of the cross-sections. This has been observed in many experiments, supporting the QCD theory.

The value of the QCD coupling constant  $\alpha_s = \frac{g_s^2}{4\pi}$  depends on the energy. One has found in the analysis of the scaling violations:

$$\alpha_s(M_z^2) \approx 0.118 \pm 0.011\tag{2}$$

( $M_z$ : mass of the  $Z$ -boson,  $M_z \cong 91.2$  GeV).

One can express  $\alpha_s(\mu)$  as a function of the QCD scale parameter  $\Lambda_c$ :

$$\begin{aligned}\alpha_s(\mu)^{-1} &\approx \left(\frac{\beta_0}{4\pi}\right) \ln\left(\frac{\mu^2}{\Lambda_c^2}\right) \\ \beta_0 &= \left(11 - \frac{2}{3} n_f\right)\end{aligned}\tag{3}$$

The experiment give:

$$\Lambda_c \approx 217_{-23}^{+25} \text{ MeV} .\tag{4}$$

The standard electroweak model is based on the gauge group  $SU(2) \times U(1)$ . There are three  $W$ -bosons, related to the  $SU(2)$  group, and one  $B$ -boson, related to the  $U(1)$ -group. The lefthanded quarks and leptons are  $SU(2)$ -doublets, the righthanded ones are singlets. Thus parity is violated maximally.

The gauge invariance of the  $SU(2) \times U(1)$ -model is broken spontaneously by the "Higgs"-mechanism[3]. The masses for the gauge bosons are generated by the spontaneous symmetry breaking. Goldstone bosons end up as longitudinal components of the gauge bosons. In the "Higgs" mechanism there exists a self-interacting complex doublet of scalar fields. In the process of symmetry breaking the neutral component of the scalar doublet acquires a vacuum expectation value  $v$ , which is directly related to the Fermi constant of the weak interactions. Thus the vacuum expectation value is known from the experiments, if the theory is correct:

$$v \cong 246 \text{ GeV}\tag{5}$$

This energy sets the scale for the symmetry breaking. Three massless Goldstone bosons are generated, which are absorbed to give masses to the  $W^+, W^-$  and  $Z$ -bosons. The remaining component of the complex doublet is the "Higgs"-boson, thus far a hypothetical particle. It would be the elementary scalar boson in the Standard Model. One hopes to find this particle with the new accelerator LHC at CERN, which starts to operate in 2008.

In the  $SU(2) \times U(1)$  model there are two neutral gauge bosons, which are mixtures of  $W_3$  and  $B$ , the  $Z$ -boson and the photon. The associated mixing angle  $\Theta_w$  is a fundamental parameter which has to be fixed by experiment.

It is given by the  $Z$ -mass, the Fermi constant and  $\alpha$ :

$$\sin^2 \Theta_w \cdot \cos^2 \Theta_w = \frac{\pi \alpha (M_Z)}{\sqrt{2} \cdot G_F \cdot M_Z^2}. \quad (6)$$

Using  $\alpha (M_Z)^{-1} = 128,91 \pm 0.02$ , one finds:

$$\sin^2 \Theta_w = 0.23108 \pm 0.00005. \quad (7)$$

Note that  $\sin^2 \Theta_w$  is also related to the mass ratio  $M_W/M_Z$ :

$$\begin{aligned} \sin^2 \Theta_w &= 1 - M_W^2/M_Z^2 \\ M_Z &= M_W/\cos \Theta_w. \end{aligned} \quad (8)$$

In the Standard Model the interactions depend on 28 fundamental constants, e. g. on the finestructure constant  $\alpha$ . There were many attempts to get further insights, how these constants arise and whether one might calculate them, but thus far all without success. At the moment we can only determine these constants by the experiments.

In physics we are dealing with the laws of nature, but little thought is given to the boundary condition of the universe, related directly to the Big Bang. We do not know at the moment, what role is played by the fundamental constants, but these constants could form a bridge between these boundary conditions and the local laws of nature. Thus they would be relicts of the Big Bang.

What are the fundamental constants? Some physicists believe that at least some of them are just cosmic accidents, fixed by the dynamics of the Big Bang. If the Big Bang would be repeated, these constants would take different values. Thus the constants are arbitrary, depending on details of the Big Bang. Obviously in this case there is no way to calculate the fundamental constants.

Indeed, some fundamental constants might be cosmic accidents, but it is unlikely, that this is the case for all the 28 fundamental constants. New interactions, discovered e. g. with the new LHC-accelerator at CERN, might offer a way to calculate at least some of the fundamental constants.

We also do not understand, why the fundamental constants are constant in time. Small time variations are indeed possible and even suggested by astrophysical experiments. In the theory of superstrings one expects time variations of the fundamental constants, in particular of the finestructure constant, of the QCD scale parameter, and of the weak interaction coupling constants.

If one finds that the fundamental constants are changing in time, then they are not just numbers, set into the fabric of basic physics – rather they would be dynamical quantities which change according to some deeper laws that we have to understand. These laws would be truly fundamental and may even point the way to a unified theory including gravity.

## 2 Fundamental Constants in the Standard Model

The Standard Model is the successful theory of the observed particle physics phenomena. However, the electroweak interactions and QCD depend on 28 fundamental constants, and within the Standard Model there is no way to calculate these constants or to derive relations between them.

The most famous fundamental constant is the finestructure constant  $\alpha$ , introduced in 1916 by Arnold Sommerfeld:

$$\alpha = \frac{e^2}{\hbar c} . \quad (9)$$

In this constant the electromagnetic coupling  $e$  enters, as well as the Planck constant  $\hbar$ , the constant of the quantum physics, and the speed of light  $c$ , the basic constant of the theory of relativity. Sommerfeld realized that  $\alpha$  is a dimensionless number, close to the inverse of the prime number 137. The experiments give the following value for  $\alpha^{-1}$ : 137,03599907(9).

As early as 1905 Max Planck pointed out in a letter to Paul Ehrenfest that  $h$  and  $\frac{e^2}{c}$ , both having the physical dimension of an action, were roughly of the same magnitude.

Adams and Lewis[4] proposed in 1914 a relation between  $h$ ,  $e^2$  and  $c$ :

$$h = \frac{e^2}{c} \left( \frac{8\pi^5}{15} \right)^{1/3} . \quad (10)$$

This can be rewritten as follows:

$$\alpha^{-1} = \left( \frac{8\pi^5}{15} \right)^{1/3} . \quad (11)$$

The numerical value for  $\alpha^{-1}$  is 137,348, which is rather close to the modern value 137,036.

Heisenberg proposed in 1936:

$$\alpha^{-1} = 2^{-4} 3^{-3} \pi , \quad (12)$$

which gives  $\alpha^{-1} = 137, 51$ .

In 1971 Wyler[5] published the following expression for  $\alpha$ ,

$$\alpha = \frac{9}{8\pi^4} \left( \frac{\pi^5}{2^4 \cdot 5!} \right)^{1/4} , \quad (13)$$

which gives  $\alpha^{-1} = 137, 03608$  and agrees well with the experimental value.

Richard P. Feynman wrote about the finestructure constant[6]: "It has been a mystery ever since it was discussed more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: is it related to  $\pi$  or perhaps to the base of the natural logarithms? Nobody knows. It's one of the greatest mysteries of physics: a magic number that comes to us with no understanding by man ...".

Based on  $\alpha$  the theory of Quantum Electrodynamics (QED) was developed, merging electrodynamics, quantum theory and relativity theory. It is the most successful theory in science, tested with a precision of  $10^{-7}$ .

In quantum field theory the number, describing the strength of the interactions, is not a fixed constant, but a function of the energy involved. This can be easily understood. The groundstate of a system, described by a quantum field theory, is filled with virtual pairs of quanta, e. g. with  $e^+e^-$ -pairs in QED. Thus an electron is surrounded by  $e^+e^-$ -pairs. The virtual electrons are repelled by the electrons, the virtual positrons are attracted, and the electron charge is partially shielded by the virtual positrons. Thus at relatively large distances (larger than the Compton wavelength of an electron  $\lambda_c = \frac{h}{m_e c} \approx 2.43 \cdot 10^{-12} m$ ) the electron charge is smaller than at distances less than  $\lambda_c$ . The dependence on the energy is described by the renormalization group equations, first considered by Murray Gell-Mann and Francis Low[7]:

$$\frac{d}{d \ln(q/M)} e(q) = \beta(e) , \quad (14)$$

where

$$\beta(e) = \frac{e^3}{12\pi^2} + \dots \quad (15)$$

In QED at high energies one has to include not only virtual  $e^+e^-$ -pairs, but also the  $\mu^+\mu^-$ - and  $\tau^+\tau^-$ -pairs, as well as the quark-antiquark-pairs. Doing so, one finds that the finestructure constant  $\alpha$  at the mass of the  $Z$ -boson should be the inverse of 127.8, in very good agreement with the experimental data taken with the LEP-accelerator.

Another fundamental parameter of the Standard Model is the proton mass. In the theory of the strong interaction QCD the proton mass is a parameter, which can be calculated as a function of the QCD scale parameter  $\Lambda_c$  and of the light quark masses. The QCD scale parameter has been determined by the experiments:

$$\Lambda_c = 217 \pm 25 \text{ MeV} . \quad (16)$$

The QCD-Theory gives a very clear picture of mass generation. In the limit, where the quark masses are set to zero, the nucleon mass is just the confined field energy of the gluons and quarks and can be written as:

$$M(\text{Nucleon}) = \text{const.} \cdot \Lambda_c . \quad (17)$$

The *const.* has been calculated using the lattice approach to QCD. It is about 3,9, predicting a nucleon mass in the limit  $m_q = 0$  of about 860 MeV. The observed nucleon mass (about 940 MeV) is higher, due to the contributions of the mass terms of the light quarks  $u, d, s$ , which in reality are not massless.

The mass of the proton can be decomposed as follows:

$$M_p = \text{const.} \cdot \Lambda_c + \langle p | m_u \bar{u}u | p \rangle + \langle p | m_d \bar{d}d | p \rangle + \langle p | m_s \bar{s}s | p \rangle + c_{\text{elm}} \cdot \Lambda_c . \quad (18)$$

The last term describes the electromagnetic self-energy of the proton. This self-energy is also proportional to the QCD-scale  $\Lambda$ . Calculations give:

$$c_{\text{elm}} \cdot \Lambda_c \approx 2.0 \text{ MeV} . \quad (19)$$

The up-quark mass term contributes about 20 MeV to the proton mass, according to the chiral perturbation theory, the d-quark mass term about 19 MeV. Thus the  $d$ -contribution to the proton mass is about as large as the  $u$ -contribution, although there are two  $u$ -quarks in the proton, and only one  $d$ -quark. This originates from the fact that the  $d$ -mass is larger than the  $u$ -mass.

In chiral perturbation theory the  $u$ - and  $d$ -masses can be estimated:

$$\begin{aligned} m_u &\approx 3 \pm 1 \text{ MeV} \\ m_d &\approx 6 \pm 1.5 \text{ MeV} . \end{aligned} \quad (20)$$

These masses are normalized at the scale  $\mu = 2 \text{ GeV}$ . Note that quark masses are not the masses of free particles, but of the confined quarks. The quark masses depend on the energy scale  $\mu$ , relevant for the discussion.

The mass of the strange quark can also be estimated in the chiral perturbation theory. One finds for  $\mu = 2 \text{ GeV}$ :

$$m_s \approx 103 \pm 20 \text{ MeV} . \quad (21)$$

Thus the mass of the strange quark is about 20 times larger than the  $d$ -mass. Although there are no valence  $s$ -quarks in the proton, the numerous  $\bar{s}s$ -pairs contribute about 35 MeV to the proton mass, i. e. more than the  $\bar{u}u$ - or  $\bar{d}d$ -pairs, due to the large ratio  $m_s/m_d$ . The heavy quarks  $\bar{c}c$  and  $\bar{b}b$  do not contribute much to the proton mass.

We can decompose the proton mass as follows:

$$\begin{aligned}
M_p &= 938 \text{ MeV} \\
&= \underset{\substack{\uparrow \\ QCD}}{(862)} + \underset{\substack{\uparrow \\ u - quarks}}{(20)} + \underset{\substack{\uparrow \\ d - quarks}}{(19)} + \underset{\substack{\uparrow \\ s - quarks}}{(35)} + \underset{\substack{\uparrow \\ QED}}{(2)} \text{ MeV}
\end{aligned} \tag{22}$$

The masses of the heavy quarks  $c$  and  $b$  can be estimated by considering the spectra of the particles, containing  $c$ - or  $b$ -quarks, e. g. the charm-mesons or the  $B$ -mesons. One finds:

$$\begin{aligned}
m_c(m_c) &\approx 1,24 \pm 0.09 \text{ GeV} \\
m_b(m_b) &\approx 4,2 \pm 0.07 \text{ GeV}
\end{aligned} \tag{23}$$

The dark corner of the Standard Model is the sector of the fermion masses. There are the six quark masses, the three charged fermion masses, the three neutrino masses, the four flavor mixing parameters of the quarks and the six flavor mixing parameters of the leptons (if neutrinos are Majorana particles). Altogether these parameters make up 22 of the 28 fundamental constants. The remaining six constants are  $\Lambda_c, \alpha_2, \alpha_1$ , the mass of the  $W$ -boson, the mass of the hypothetical "Higgs"-boson and the constant  $G$  for gravity.

What are the fermion masses? We do not know. They might also be due to a confined field energy, but in this case the quarks and leptons would have to have a finite radius, as e. g. in composite models. The masses would be generated by a new interaction. The experiments give a limit on the internal radius of the leptons and quarks, which is of the order of  $10^{-17}$  cm.

In the Standard Model the masses of the leptons and quarks are generated spontaneously, like the  $W$  and  $Z$ -masses. Each fermion couples with a certain strength to the scalar "Higgs"-boson via a Yukawa coupling. The fermion mass is then given by:

$$m(fermion) = g \cdot V, \tag{24}$$

where  $V$  is the vacuum expectation value of the "Higgs"-field. For the electron this Yukawa coupling constant must be very small, since  $V$  is about 246 GeV:

$$g(electron) = 0,00000208. \tag{25}$$

Nobody understands, why this constant is so tiny. The problem of fermion masses remains to be solved. It seems to be the most fundamental problem we are facing at the present time.

If one is interested only in stable matter, as e. g. in solid state physics, only seven fundamental constants enter:

$$G, \Lambda, \alpha, m_e, m_u, m_d, m_s. \tag{26}$$

The mass of the  $s$ -quark has been included, since the  $(\bar{s}s)$ -pairs contribute to the nucleon mass about 40 MeV. These seven constants describe all atoms and stable nuclei.

It is possible, that there exist relations between the fundamental constants. Relations, which seem to work very well, are the relations between the flavor mixing angles and the quark masses, which were predicted some time ago[8]:

$$\begin{aligned}
\Theta_u &= \sqrt{m_u/m_c} \\
\Theta_d &= \sqrt{m_d/m_s}.
\end{aligned} \tag{27}$$

Similar relations also exist for the neutrino masses and the associated mixing angles[9].

These relations are obtained if both for the  $u$ -type and for the  $d$ -type quarks the following mass matrices are relevant (texture 0 matrices):

$$M = \begin{pmatrix} 0 & A & 0 \\ A^* & C & B \\ 0 & B^* & D \end{pmatrix}. \quad (28)$$

It would be interesting to know whether these mass matrices are indeed realized in nature. Relations like the ones discussed above would reduce the number of fundamental constant from 28 to about 20.

### 3 Does the Finestructure Constant depend on Time?

Recent observations in astrophysics[10] indicate that the finestructure constant  $\alpha$  is a slow function of time. Billions of years ago it was smaller than today. A group of researchers from Australia, the UK and the USA analysed the spectra of distant quasars, using the Keck telescope in Hawaii. They studied about 150 quasars, some of them about 11 billion lightyears away. The redshifts of these objects varied between 0.5 and 3.5. This corresponds to ages varying between 23% and 87% of the age of our universe.

They used the "many multiplet method" and in particular studied the spectral lines of iron, nickel, magnesium, zinc and aluminium. It was found that  $\alpha$  changes in time:

$$\frac{\Delta\alpha}{\alpha} = (-0.72 \pm 0.18) \cdot 10^{-5}. \quad (29)$$

Taking into account the ages of the observed quasars, one concludes that in a linear approximation the absolute magnitude of the relative change of  $\alpha$  must be:

$$\left| \frac{d\alpha/dt}{\alpha} \right| \approx 1.2 \cdot 10^{-15}/year. \quad (30)$$

We like to mention that recent observations of quasar spectra, performed by different groups, seem to rule out a time variation of  $\alpha$  at the level given above[11].

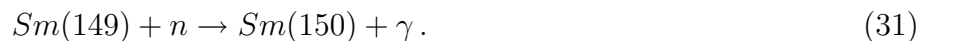
The idea that fundamental constants are not constant in time, but have a cosmological time dependence, is not new. In the 1930s P. Dirac[12] discussed a time variation of Newtons constant  $G$ . Dirac argued that  $G$  should vary by about a factor 2 during the lifetime of the universe. The latter is now fixed to about  $1.4 \cdot 10^{10}$ . Only a few years ago it was found that a possible time variation of  $G$  must be less than  $10^{-11}$  per year, and Dirac's hypothesis is now excluded.

In the 1950s L. Landau discussed a possible time variation of the finestructure constant  $\alpha$  in connection with the renormalization of the electric charge[13].

In the 1970s French nuclear physicists discovered that about 1.8 billion years ago a natural reactor existed in Gabon, West-Africa, close to the river Oklo. About 2 billion years ago uranium -235 was more abundant than today (about 3,7%). Today it is only 0,72%. The water of the river Oklo served as a moderator for the reactor. The natural reactor operated for about 100 million years.

The isotopes of the rare earths, for example the element Samarium, were produced by the fission of uranium. The observed distribution of the isotopes today is consistent with the calculation, assuming that the isotopes were exposed to a strong neutron flux.

Especially the reaction of Samarium with a neutron is interesting:



It was known that the very large cross-section for this reaction (about 60...90 kb) is due to a nuclear resonance just above threshold. The energy of this resonance is very small:  $E = 0.0973$  eV. The

position of this resonance cannot have changed in the past 2 billion years by more than 0.1 eV. Suppose  $\alpha$  has changed during this time. The energy of the resonance depends also on the strength of the electromagnetic interaction. One concludes:

$$\frac{\alpha(Oklo) - \alpha(now)}{\alpha(now)} < 10^{-7}. \quad (32)$$

Taking into account the two billion years, the relative change of  $\alpha$  per year must be less than  $10^{-16}$  per year, as estimated by T. Damour and F. Dyson[14]. This conclusion is correct only if no other fundamental parameters changed in the past two billion years. If other parameters, like the strong interaction coupling constant, changed also, the constraint mentioned above does not apply.

The Oklo constraint for  $\alpha$  is not consistent with the astrophysical observation for the relative changes of  $\alpha$  of order  $10^{-15}$  per year. No problem exists, however, if other fundamental "constants" also changed. Below we shall see that this was indeed the case.

Recently one has also found a time change of the mass ratio

$$\mu = \frac{M(proton)}{m(electron)}. \quad (33)$$

One observed the light from a pair of quasars, which are 12 billion light years away from the earth[15]. The light was emitted, when the universe was only 1.7 billion years old. The study of the spectra revealed, that the ratio  $\mu$  has changed in time:

$$\frac{\Delta\mu}{\mu} \approx (2 \pm 0.6) \cdot 10^{-5}. \quad (34)$$

Taking into account the lifetime of 12 billion years, the change of  $\mu$  per year in a linear approximation would be  $10^{-15}$  / year. We shall return to this time change of  $\mu$  later.

## 4 Grand Unification

In the Standard Model we have three basic coupling constants, one for QCD ( $\alpha_3 = \alpha_s$ ) and two for the electroweak interactions, based on the gauge group  $SU(2) \times U(1)$ :  $\alpha_2$  and  $\alpha_1$ . The gauge group of the Standard Model is  $SU(3)_c \times SU(2) \times U(1)$ . The three gauge interactions are independent of each other.

Since 1974 the idea is discussed that the gauge group of the Standard Model is a subgroup of a larger simple group, and the three gauge interactions are embedded in a Grand Unified Theory (GUT). Such a Grand Unification implies that  $\alpha_3, \alpha_2$  and  $\alpha_1$  are related and can be expressed in terms of the unified coupling constant  $\alpha_{un}$  and the energy scale of the unification  $\Lambda_u$ .

The simplest theory of Grand Unification is a theory based on the gauge group  $SU(5)$ [16]. The quarks and leptons of one generation can be described by two  $SU(5)$ -representations. Let us consider the 5-representation of  $SU(5)$ . After the breakdown of  $SU(5)$  to  $SU(3) \times SU(2) \times U(1)$  we obtain:

$$\begin{aligned} 5 &\rightarrow (3, 1) + (1, 2) \\ \bar{5} &\rightarrow (\bar{3}, 1) + (1, 2). \end{aligned} \quad (35)$$

Thus the 5-representation contains a color triplet, which is a singlet under  $SU(2)$ , and a color singlet, which is a  $SU(2)$ -doublet. We can identify the associated particles, neglecting the flavor mixing, and obtain for the first lepton-quark family:

$$(5) = \begin{pmatrix} \bar{d}_r \\ \bar{d}_g \\ \bar{d}_b \\ \nu_e \\ e^- \end{pmatrix}. \quad (36)$$



The representation with the next higher dimension is the 10–representation, which is an antisymmetric second–rank tensor. The 10–representation decomposes after the symmetry breaking as follows:

$$(10) \rightarrow (5, 1) = (3, 2) + (1, 1) \quad (37)$$

In terms of the lepton and quark fields of the first generation we can write the 10–representation (a  $5 \times 5$ –matrix) as follows:

$$(10) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{u}_b & -\bar{u}_g & -\bar{u}_r & -\bar{d}_r \\ -\bar{u}_b & 0 & \bar{u}_r & \bar{u}_g & -\bar{d}_g \\ \bar{u}_g & -\bar{u}_r & 0 & -\bar{u}_b & -\bar{d}_b \\ u_r & u_g & u_b & 0 & e^+ \\ d_r & d_g & d_b & -e^+ & 0 \end{pmatrix}. \quad (38)$$

Combining these two representations, one finds precisely the lepton and quarks, forming one generation:

$$\bar{5} + 10 \rightarrow (3, 2) + 2(\bar{3}, 1) + (1, 2) + (1, 1). \quad (39)$$

For the first generation one has:

$$\bar{5} + 10 \rightarrow \begin{pmatrix} u \\ d \end{pmatrix}_L + \bar{u}_L + \bar{d}_L + \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L + e_L^+. \quad (40)$$

The second and third generation are analogous. The unification based on the gauge group  $SU(5)$  has a number of interesting features:

- 1) The electric charge is quantized.

$$trQ = 0 \rightarrow Q(d) = \frac{1}{3} Q(e^-) \quad (41)$$

- 2) At some high mass scale  $\Lambda_{un}$  the gauge group of the Standard Model turns into the group  $SU(5)$ , and there is only one single gauge coupling. The three coupling constants  $g_3, g_2, g_1$  for  $SU(3), SU(2)$  and  $U(1)$  must then be of the same order of magnitude, related to each other by constants.

The rather different values of the coupling constants  $g_3, g_2, g_1$  at low energies must be due to renormalization effects. This would also give a natural explanation of why the strong interactions are strong and the weak interactions are weak. It has to do with the size of the corresponding group.

Apart from normalization constants the three coupling constants  $g_3, g_2$  and  $g_1$ , are equal at the unification mass  $\Lambda_{un}$ . Thus the  $SU(2) \times U(1)$  mixing angle, given by  $\tan\Theta_w = \frac{g_1}{g_2}$ , is fixed at or above  $\Lambda_{un}$ :

$$\sin^2\Theta_w = trT_3^2/trQ^2 = \frac{3}{8}. \quad (42)$$

At an energy scale  $\mu \ll \Lambda_{un}$  the parameter  $\sin^2\Theta$  changes along with the three coupling constants:

$$\begin{aligned} \frac{\sin^2\Theta_w}{\alpha} - \frac{1}{\alpha_s} &= \frac{11}{6\pi} \ln\left(\frac{M}{\mu}\right) \\ \alpha/\alpha_s &= \frac{3}{10} (6\sin^2\Theta_w - 1). \end{aligned} \quad (43)$$

At  $\mu = M_z$  the electroweak mixing angle has been measured:  $\sin^2\Theta_w(M_u) \cong 0.2312$ . Note that above the unification energy  $\alpha$  and  $\alpha_s$  are related:

$$\alpha/\alpha_s = 3/8. \quad (44)$$

This relation can be checked by experiment. In order to get a rough agreement between the observed values for  $g_3, g_2$  and  $g_1$  and the values predicted by the  $SU(5)$  theory, one can easily see that the unification scale must be very high. Note that

$$\begin{aligned} \ln\left(\frac{M}{\mu}\right) &= \frac{6\pi}{11} \left( \frac{\sin^2\Theta_w}{\alpha} - \frac{1}{\alpha_s} \right) \\ \mu &= M_Z \\ \ln(M/M_Z) &\cong 39,9 \\ M &\approx 2 \cdot 10^{15} \text{ GeV}. \end{aligned} \quad (45)$$

However, the precise values of the three coupling constants, determined by the LEP-experiments, disagree with the  $SU(5)$  prediction. The three coupling constants do not converge at high energies to a single coupling constant  $\alpha_{un}$ . However, a convergence takes place, if supersymmetric particles are added. Supersymmetry implies that for each fermion a boson is added (s-leptons, s-quarks), and for each boson a new fermion is introduced (photino, etc.). These new particles are not observed in the experiments. It is assumed that they have a mass of about 1 TeV or more.

The new particles contribute to the renormalization of the gauge coupling constants at high energies (about 1 TeV), and a convergence of the three coupling constants taken place. Thus a supersymmetric version of the  $SU(5)$ -theory is consistent with the experiments.

In theories of Grand Unification like the  $SU(5)$ -theory one has typically quarks, antiquarks and leptons in one fermion representation. This implies that the proton can decay, e. g.  $p \rightarrow e^+\pi^0$ . The lifetime depends on the mass scale for the unification. For  $\Lambda_{un} = 5 \cdot 10^{14}$  GeV in the  $SU(5)$ -theory without supersymmetry one finds  $10^{30}$  years for the proton lifetime. The experimental lower limit is about  $10^{31}$  years.

There is a natural embedding of a group  $SU(n)$  into  $SO(2n)$ , corresponding to the fact that  $n$  complex numbers can be represented by  $2n$  real numbers. Thus one may consider to use the gauge group  $SO(10)$  instead of  $SU(5)$ . This was discussed first in 1975 by P. Minkowski and the author[17]. The fermions of one generation are described by a 16-dimensional spinor representation of  $SO(10)$ .

Since  $SU(5)$  is a subgroup of  $SO(10)$ , one has the following decomposition:

$$16 \rightarrow \bar{5} + 10 + 1. \quad (46)$$

Thus the fermions of the  $SU(5)$ -theory are obtained, plus one additional fermion (per family). This state is an  $SU(5)$ -singlet and describes a lefthanded antineutrino field. Using the leptons and quarks of the first generation we can write the 16-representation as follows in terms of lefthanded fields:

$$(16) = \begin{pmatrix} \bar{\nu}_e & \bar{u}_r & \bar{u}_g & \bar{u}_b & : & u_r & u_g & u_b & \nu_e \\ e^+ & \bar{d}_r & \bar{d}_g & \bar{d}_b & : & d_r & d_g & d_b & e^- \end{pmatrix} \quad (47)$$

An interesting feature of the  $SO(10)$ -theory is that the gauge group for the electroweak interactions is larger than in the  $SU(5)$ -theory.  $SO(10)$  has the subgroup  $SO(6) \times SO(4)$ . Since  $SO(4)$  is isomorphic into  $SU(2) \times SU(2)$ , one finds:

$$SO(10) \rightarrow SU(4) \times SU(2)_L \times SU(2)_R. \quad (48)$$

The group  $SU(4)$  must contain the color group  $SU(3)^c$ . The 16-representation of the fermions decomposes under  $SU(4)$  into two 4-representations. These contain three quarks and one lepton, e. g.  $(d_r, d_g, d_b)$  and  $e^-$ . One can interpret the leptons as the fourth color. However, the gauge group  $SU(4)$  must be broken at high energies:

$$SU(4) \rightarrow SU(3) \times U(1). \quad (49)$$

We obtain at low energies the gauge group

$$SU(3)^c \times SU(2)_L \times SU(2)_R \times U(1). \quad (50)$$

However, the masses of the gauge bosons for the group  $SU(2)_R$  must be much larger than the observed  $W$ -bosons, related to the group  $SU(2)_L$ .

In the  $SU(5)$ -theory the minimal number of fermions of the Standard Model is included. But in the  $SO(10)$ -theory a new righthanded neutrino is added (the  $\bar{\nu}_{eL}$ -particle). This righthanded fermion is interpreted as a heavy Majorana particle. In this case a mass for the lefthanded neutrino is generated by the "see-saw"-mechanism[18]. Thus in the  $SO(10)$ -theory the neutrinos are massive, while in the  $SU(5)$ -theory they must be massless.

The  $SO(10)$ -theory is much more symmetrical than the  $SU(5)$ -theory, and it is hard to believe that Nature would stop at  $SU(5)$ , if Nature has chosen to unify the basic interactions.

In the  $SO(10)$ -theory there is one additional free parameter, related to the masses of the righthanded  $W$ -bosons. Since righthanded charged currents are not observed, the masses of the associated  $W$ -bosons must be rather high, at least 300 GeV. Since there is this new parameter  $M_R$  in the  $SO(10)$ -theory, it can be chosen such that the coupling constant converges at very high energies, without using supersymmetric particles. If one chooses  $M_R \sim 10^9 \dots 10^{11}$  GeV, the convergence occurs.

The idea of Grand Unification leads to the reduction of the fundamental constants by one. The three gauge coupling constants of the Standard Model can be expressed in terms of a unified coupling constant  $\alpha_u$  at the energy  $\Lambda_u$ , where the unification takes place. The three coupling constants are replaced by  $\alpha_u$  and  $\Lambda_u$ .

In a Grand Unified Theory the three coupling constants of the Standard Model are related to each other. If e. g. the finestructure constant shows a time variation, the other two coupling constants should also vary in time. Otherwise the unification would not be universal in time. Knowing the time variation of  $\alpha$ , one should be able to calculate the time variation of the other coupling constants.

We shall investigate only the time change of the QCD coupling constant  $\alpha_s$ . We can also calculate the time change of the weak coupling constant, but this would be useless, since we have no information about the weak coupling constant billions of years ago, at least not about small changes of the order of  $10^{-4}$  or less.

We use the supersymmetric  $SU(5)$ -theory to study the time change of the coupling constants[19, 20]. The change of  $\alpha$  is traced back to a change of the unified coupling constant at the energy of unification and to a change of the unification energy. These changes are related to each other:

$$\frac{1}{\alpha} \frac{\dot{\alpha}}{\alpha} = \frac{8}{3} \cdot \frac{1}{\alpha_s} - \frac{10}{\pi} \frac{\dot{\Lambda}_{un}}{\Lambda_{un}}. \quad (51)$$

We consider the following three scenarios:

- 1)  $\Lambda_{un}$  is kept constant,  $\alpha_u = \alpha_u(t)$ .

Then we obtain:

$$\frac{1}{\alpha} \frac{\dot{\alpha}}{\alpha} = \frac{8}{3} \frac{1}{\alpha_s} \frac{\dot{\alpha}_s}{\alpha_s}. \quad (52)$$

Using the experimental value  $\alpha_s(M_Z) \approx 0.121$ , we find for the time variation of the QCD scale[19]:

$$\frac{\dot{\Lambda}}{\Lambda} \approx R \cdot \frac{\dot{\alpha}}{\alpha}$$

$$R \approx 38 \pm 6. \quad (53)$$

The uncertainty in  $R$  comes from the uncertainty in the determination of the strong interaction coupling constant  $\alpha_s$ .

A time variation of the QCD scale  $\Lambda$  implies a time change of the proton mass and of the masses of all atomic nuclei. The change of the nucleon mass during the last 10 billion years amounts to about 0.3 MeV.

In QCD the magnetic moments of the nucleon and of the atomic nuclei are inversely proportional to the QCD scale parameters  $\Lambda$ . Thus we find for the nuclear magnetic moments:

$$\frac{\dot{\mu}}{\mu} = \frac{\frac{d}{dt}\left(\frac{1}{\Lambda}\right)}{\frac{1}{\Lambda}} = -\frac{\dot{\Lambda}}{\Lambda} = -R \cdot \frac{\dot{\alpha}}{\alpha}. \quad (54)$$

Taking the astrophysics result for  $(\dot{\alpha}/\alpha)$ , we would obtain:

$$\frac{\dot{\Lambda}}{\Lambda} \approx 4 \cdot 10^{-14}/yr. \quad (55)$$

2) The unified coupling constant is kept invariant, but  $\Lambda_{un}$  changes in time. In that case we find[22]:

$$\frac{\dot{\alpha}}{\alpha} \cong -\alpha \cdot \frac{10}{\pi} \frac{\dot{\Lambda}_{un}}{\Lambda_{un}} \quad (56)$$

and

$$\frac{\dot{\Lambda}}{\Lambda} \approx -31 \cdot \frac{\dot{\alpha}}{\alpha}. \quad (57)$$

The change of the unification mass scale  $\Lambda_{un}$  can be estimated, taking as input the time variation of the finestructure constant  $\alpha$ . One finds that this mass was 8 billion years ago about  $8 \cdot 10^{12}$  GeV higher than today. In a linear approximation  $\Lambda_{un}$  is decreasing at the rate

$$\dot{\Lambda}_{un}/\Lambda_{un} \approx -7 \cdot 10^{-14}/yr. \quad (58)$$

The relative changes of  $\Lambda$  and  $\alpha$  are opposite in sign. While  $\alpha$ , according to ref. [10], is increasing with a rate of  $10^{-15}/yr$ , the QCD scale  $\Lambda$  and the nucleon mass are decreasing with a rate of about  $3 \cdot 10^{-14}/yr$ . The magnetic moments of the nucleons and of nuclei would increase:

$$\frac{\dot{\mu}}{\mu} \approx 3 \cdot 10^{-14}/yr. \quad (59)$$

3) The third possibility is that both  $\alpha_u$  and  $\Lambda_{un}$  are time-dependent. In this case we find:

$$\frac{\dot{\Lambda}}{\Lambda} \cong 46 \cdot \frac{\dot{\alpha}}{\alpha} + 1,07 \cdot \frac{\dot{\Lambda}_{un}}{\Lambda_{un}}. \quad (60)$$

On the right two terms appear:  $(\dot{\alpha}/\alpha)$  and  $(\dot{\Lambda}_{un}/\Lambda_{un})$ . These two terms might conspire in such a way that  $(\dot{\Lambda}/\Lambda)$  is smaller than about  $(40 \cdot \dot{\alpha}/\alpha)$ .

In the first two scenarios the QCD-scale  $\Lambda$  changes faster than the finestructure constant. In the third scenario the change of the QCD-scale depends both on the change of  $\alpha$  and on the change of the unification scale. The signs of both changes can be different, and one could have a partial cancellation. In this case the change of  $\Lambda$  could be less than in the first two scenarios. This possibility will be discussed later.

The question arises, whether a time change of the QCD scale parameter could be observed in the experiments. The mass of the proton and the masses of the atomic nuclei as well as their magnetic moments depend linearly on the QCD scale. If this scale changes, the mass ratio  $M_p/m_e = \mu$  would change as well, if the electron mass remains constant.

We mentioned before that the mass ratio  $\mu$  seems to show a time variation – in a linear approximation one has about

$$\frac{\Delta\mu}{\mu} \approx 10^{-15}/year. \quad (61)$$

If we take the electron mass to be constant in time, this would imply that the QCD-scale  $\Lambda$  changes with the rate

$$\frac{\Delta\Lambda}{\Lambda} \approx 10^{-15}/year. \quad (62)$$

The connection between a time variation of the finestructure constant and of the QCD scale, discussed above, is only valid, if either the unified coupling constant or the unification scale depends on time, not both. If both the unification scale and the unified coupling constant are time dependent, we should use instead eqs. (51) and (60). As seen in particular in eq. (60), there might be a cancellation between the two terms, and in this case the time variation of the QCD-scale would be smaller than  $10^{-14}/year$ . If the two terms cancel exactly, the QCD-scale would be constant, but this seems unlikely. Thus a time variation of the QCD-scale of the order of  $10^{-15}/year$  (see eq. (62)) is quite possible.

Can such a small time variation be observed in experiments here on the earth? In Quantum Optics one can carry out very precise experiments with lasers. In the next chapter we shall describe such an experiment at the Max-Planck-Institute of Quantum Optics in Munich, which was designed especially to find a time variation of the QCD scale  $\Lambda$  as expected by the Grand Unified Theory.

## 5 Results from QuantumOptics

Being the simplest of all stable atoms, the hydrogen atom is a very good test object for checking fundamental theories. Its atomic properties can be calculated with unprecedented accuracy. But at the same time the level structure of the hydrogen atom can be very accurately probed, using spectroscopy methods in the visible, infrared and ultraviolet regions. Thus the hydrogen atom plays an important rôle in determining the fundamental constants like the finestructure constant.

Measurements of the Lamb shift and the 2S hyperfine structure permit very sensitive tests of quantum electrodynamics. Combining optical frequency measurements in hydrogen with results from other atoms, stringent upper limits for a time variation of the finestructure constant[21] and of the QCD scale parameter can be derived.

The employment of frequency combs turned high-precision frequency measurements into a routine procedure. The unprecedented accuracy of the frequency comb have opened up wide perspectives for optical atomic clock applications in fundamental physics. Frequency measurements in the laboratory have become competitive recently in terms of sensitivity to a possible time variation of the fine-structure constant in the present epoch. Though the time interval covered by these

measurements is restricted to a few years, very high accuracy compensates for this disadvantage. Their sensitivity becomes comparable with astrophysical and geological methods operating on a billion-year time scale.

Important advantages of the laboratory experiments are: The variety of different systems that may be tested, the possibility to change parameters of the experiments in order to control systematic effects, and the determination of the drift rates from the measured data. Modern precision frequency measurements deliver information about the stability of the present values of the fundamental constants, which can only be tested with laboratory measurements. At the same time only non-laboratory methods are sensitive to processes that happened in the early universe, which can be much more severe as compared to the present time. As both classes of experiments probe the constants at different epochs, they supplement each other to get a more detailed view of the possible time variation of the fundamental constants.

In the experiment of the MPQ-group in Munich[22] one was able to determine the frequency of the hydrogen 1S–2S–transition to 2466061102474851(34) Hz.

A comparison with the experiment performed in 1999 gives an upper limit on a time variation of the transition frequency in the time between the two measurements, 44 months apart. One finds for the difference  $(-29 \pm 57) \text{ Hz}$ , i. e. it is consistent with zero.

One can compare this frequency with the frequency of the hyperfine transition in  $^{133}\text{Cs}$ , and one obtains for the fractional time variation:  $(-3.2 \pm 6.3) \cdot 10^{-15} \text{ yr}^{-1}$ .

Comparing the 1S – 2S hydrogen transition with the hyperfine transition of Cesium  $^{133}\text{Cs}$ , one can obtain information about the time variation of the ratio  $\alpha/\alpha_s$ . This is the case, since the Cesium hyperfine transition depends on the magnetic moment of the Cesium nucleus, and the magnetic moment is proportional to  $(1/\Lambda_c)$  ( $\Lambda_c$ : QCD scale parameter). If  $\Lambda_c$  varies in time, the magnetic moment will also vary.

One has also found interesting limits[22] on the time variation of  $\alpha$ :

$$-1.5 \cdot 10^{-15} / \text{yr} < \frac{\delta\alpha}{\alpha} < 0.4 \cdot 10^{-15} / \text{yr}. \quad (63)$$

Recently one has obtained a limit for the time variation of the magnetic moment of the Cesium nucleus[23]:

$$\frac{\delta\mu}{\mu} = (1.5 \pm 2.0) \cdot 10^{-15} / \text{yr}. \quad (64)$$

These results are consistent with zero. However, the limit on the time variation of  $\alpha$  is of the same order as the astrophysics result.

The result concerning the magnetic moment implies a limit on the time variation of  $\Lambda_c$ :

$$\frac{\Delta\Lambda_c}{\Lambda_c} = (-1.5 \pm 2.0) \cdot 10^{-15} / \text{yr}. \quad (65)$$

This result is in disagreement with our results, based on the assumption, that either  $\alpha_u$  or  $\Lambda_{un}$  change in time. We obtained about  $10^{-14} / \text{yr}$ , which is excluded by the experiments.

The result given above is consistent with no time change for  $\Lambda_c$ , but it also agrees with a small time change of the order of  $10^{-15}$  per year. If we assume that the electron mass does not change in time, such a change of  $\Lambda_c$  would agree with the astrophysics result on the time variation of the ratio  $M(\text{proton}) / m(\text{electron})$ . Theoretically we would expect such a time variation if both  $\Lambda_{un}$  and  $\alpha_u$  change in time, as discussed at the end of chapter 4.

## 6 Conclusions and Outlook

In this review we have summarized our present knowledge about the fundamental constants and their possible time variation. Today we do not know how these constants are generated or whether they might depend on time. The phenomenon of the fundamental constants in physics remains a mystery. There might be relations between these constants, e. g. between the flavor mixing angles and the fermion masses, or relations between the three coupling constants, implied by the idea of Grand Unification. This would reduce the number of basic constants from 28 down to a smaller number, but at least 18 fundamental constants would still exist.

A possible time variation of the fundamental constants must be rather slow, at least for those fundamental constants, which are measured very precisely, i. e. the finestructure constant, the QCD-scale  $\Lambda$ , and the electron mass. The constant of gravity  $G$  is known with a precision of  $10^{-11}$ . All other fundamental constants, e. g. the masses of the other leptons or the masses of the quarks, are not known with a high precision. We would not notice, if for example the mass of the  $\tau$ -lepton would change with a rate of  $10^{-5}$  per year, or if the mass of the  $b$ -quark would change with a rate of  $10^{-4}$  per years.

The present limits on the time variation of the finestructure constant, the QCD scale or the electron mass are of the order of  $10^{-15}$  / year. These limits should be improved by at least two orders of magnitude in the near future.

The astrophysics experiment [15] indicates a time variation of  $\Lambda_c$  of the order of  $10^{-15}/year$ . Is this experiment correct? We do not know. Other astrophysics experiments should be carried out. These experiments have a time scale of about  $10^{10}$  years. They cannot determine a time variation at the present time, 14 billion years after the Big Bang. The experiment in quantum optics, however, have a time scale of only a few years, but a very high precision.

If the astrophysics experiments indicate a time variation of the order of  $10^{-15}/year$ , it does not mean that experiments in quantum optics should also give such a time variation. It might be that until about 10 billion years after the Big Bang the constants did vary slowly, but after that they remain constant. No theory exists thus far for a time variation, and there is no reason to believe that a time variation should be linear, i. e.  $10^{-15}/year$  throughout the history of our universe. If the fundamental constants do vary, one would expect that the variation is rather large very close to the Big Bang. In the first microseconds after the Big Bang constants like  $\alpha$  or  $\Lambda_c$  might have changed by a factor 2, and we would not know.

There are limits from nucleosynthesis on possible variations of  $\alpha$  or  $\Lambda_c$ , but they are rather weak and test these constants about one minute after the Big Bang, not earlier.

In cosmology one should consider time variations of fundamental parameters in more detail. They arise naturally in the superstring theories. Perhaps allowing a suitable time variation of the constants leads to a better understanding of the cosmic evolution immediately after the Big Bang.

Some, but not all fundamental constants might simply be cosmic accidents. But in this case the constants had a rapid time variation right after the Big Bang. If this is true, we shall never be able to determine theoretically the values of those constants. But allowing time variations might lead to better cosmological theories and to a better understanding of particle physics. Particle physics and cosmology together would give a unified view on our universe.

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